1. (20 pts) Use induction to prove that $n^3 - 7n$ is divisible by 3 for all natural numbers $n \geq 0$.  
**Hint:** You might have to use $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$. 

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 3 problems in total.
2. (10 pts) Prove the following statement: If the average high temperature in New Brunswick over the past 365 days was 53° F, then there must have been a day (among the past 365 days) on which the high temperature was at least 53° F.

3. (10 + 10 = 20 pts) For each of the following statements, state whether you think the statement is True or False. If you claim that a statement is True, you must supplement your answer with a proof, and if you claim that a statement is False, you must provide a concrete (i.e., provide actual numbers) counterexample to the statement.

(a) If \( \alpha \) is an irrational number and \( \beta \) is a rational number then \( \alpha \beta \) must be irrational.

(b) The ratio of two distinct positive irrational numbers is always irrational.