Quiz III (CS 205 - Fall 2019) (Solutions)

Name:
NetID:
Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 3 problems in total.

1. (10 pts) Consider the following statement

\[ \text{If } Y \subseteq X \text{ then } \overline{X} \subseteq \overline{Y}. \]

Below is an incomplete proof of the statement. Complete the proof.

**Proof:** Assume that \( Y \subseteq X \). Recall that in order to show that \( \overline{X} \subseteq \overline{Y} \) we need to prove that

\[ a \in \overline{X} \rightarrow a \in \overline{Y}. \]

Let \( a \) be an arbitrary element of the universe such that \( a \in \overline{X} \). This is equivalent to the statement

\[ a \notin X. \]

(Now use the fact that \( a \notin X \) and \( Y \subseteq X \) to show that \( a \in \overline{Y} \). This would complete the proof)

Since \( Y \subseteq X \) and \( a \notin X \), \( a \) cannot be contained in \( Y \) and so \( a \notin Y \). This is equivalent to

\[ a \in \overline{Y}. \]

This completes the proof

2. (10 + 10 = 20 pts) For each of the following statements, state whether you think the statement is True or False and provide an explanation for your answer.

   (a) Let \( A = \{a, b, c, d, e\} \) and \( B = \{1, 2, 3, 4, 5\} \). Then it is possible to define a function \( f : A \rightarrow B \) such that \( f \) is bijective.
      **Solution:** True. Consider the function \( f : A \rightarrow B \) such that
      \[ f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4, f(e) = 5. \]
      Clearly \( f \) is a bijection.

   (b) Let \( U = \{1, 2, 3, 4, 5\} \) then there is a set \( S \subseteq U \) such that \( \emptyset \notin \text{pow}(S) \).
      **Solution:** False. Let \( S \subseteq U \) be an arbitrary subset of \( U \). Since \( \emptyset \subseteq S \), it must be the case that
      \[ \emptyset \in \text{pow}(S), \]
      and so the statement is False.
3. \textbf{(20 pts)} There are 150 students in CS 205: 120 know Python, 40 know Rust, and 30 know both Python and Rust. How many student are there in CS 205 that know either Python or Rust?

\textbf{Solution:} Let $A$ be the set of students who know Python, $B$ be the set of students who know Rust. Then $A \cap B$ is the set of students who know both Python and Rust, and $A \cup B$ is the set of students who know either Python or Rust. Using the given data, we can conclude

$$|A \cup B| = |A| + |B| - |A \cap B| = 120 + 40 - 30 = 130,$$

and so the number of students who know either Python or Rust is 130.