CS206: Practice Problems

1. Two fair dice are rolled. What is the probability that the number showing on one of them will be twice the number showing on the other?

2. An urn has 5 white, 4 black, and 3 red chips. Four chips are drawn at random, without replacement. What is the sample space? What is the probability that you got $W, R, W, B$ (this is event $A$)? What is the probability that you got at least two whites (this is event $B$)? What is $P(A|B)$?

3. Urn 1 has three red, two black, and five white chips. Urn 2 has two red, four black, and three white. One chip is drawn at random from each urn. What is the probability that both chips are the same color? Given that they are the same color, what is the probability of White?

4. How many ways can you choose a president, vice-president, and treasurer from club with 25 members? If the club is 12 men and 13 women, how many of these choices have no male officers? No male president?

5. A liquor store owner will cash checks up to 50 dollars, but is wary about customers wearing sunglasses. Fifty percent of checks written by customers wearing sunglasses bounce while only two percent of the checks written by persons not wearing sunglasses, bounce. Half the customers wear sunglasses. If a certain check bounces, what is the probability it had been written by a customer with sunglasses?

6. How many even numbers in $[100, 999]$ have distinct digits? How many palindromes (numbers that are the same when you write them backwards) are in this range? How about the range $[1000, 9999]$?

7. (a) How many rooted binary trees with 7 nodes are there? How many of them have three nodes in the left subtree? How many of them have height 3?

   (b) You choose a 7-node tree at random (equally likely probability). What is the probability its height is 3 given there are three nodes in the left subtree?

8. (a) 75 people have $5 bills and 25 people have $10 bills, and they are in line at a ticket counter which has no money and which charges $5 for admission. If a $10 bill is presented and there is no change, the line stops. What is the probability that the ticket seller always (after the first person in line, that is) has at least one $5 bill for change?

   (b) As above, what is the probability the ticket seller was always able to make change.

   (c) What is the probability that the ticket seller was always able to make change if he is now allowed to use his own, single $5 dollar bill, if needed?

9. Each of the eight letters of the word “tomorrow” are written on a sheet of paper. The sheets are then shuffled, so that each sequence of 8 letters is equally likely. What is the probability that after shuffling, the order of the sheets of paper spells the word “tomorrow” correctly?
10. Alice picks 5 cards from a deck of 52 cards. We say that there are two distinct pairs if the 5 cards contain 2 cards of a first denomination, 2 cards of a different denomination and a single card of yet another denomination.

(a) What is the probability of Alice getting two distinct pairs? Assume all 5-card hands are equally likely.
(b) Given that two of the cards alice has are of the same denomination, what is the probability that she has 2 distinct pairs?

11. Alice and Bob are playing the following game. They roll a pair of fair die. If the outcome is 6, Alice wins. If the outcome is a 3 or 5, Bob wins. Otherwise they say the roll was indecisive and keep rolling until either a 6 (Alice wins) or, an outcome of 3 or 5 is rolled (Bob wins).

What is the probability that Alice eventually wins?

12. Let $A$ and $B$ be events in sample space $S$ with the properties that $A \cap B = \emptyset$, $P(A) > 0$, $P(B) > 0$. Say if the following statements are true or false:

(a) $P(A|B^c) > P(A)$
(b) $P(B|A) \geq P(B)$
(c) $P(A^c)P(B^c) = 0$
(d) $P(A^c|B) < 1$

13. Let $A$ and $B$ be two events with $P(A) > 0$ and $P(B) > 0$. Show that if $P(A|B) \geq P(A)$, then $P(B|A) \geq P(B)$.

14. An event $B$ is said to suggest event $A$ if $P(A|B) > P(A)$ and does not suggest if $P(A|B) < P(A)$. Show that if $0 < P(B) < 1$, then the following is true: $B$ suggests $A$ if and only if $B^c$ does not suggest $A$.

15. A computer has printer ($P$), disk ($D$) and terminal ($T$) outputs. Sixty percent of all output characters are on $D$, thirty percent on $P$, and the rest on $T$. The error rate for $D$ is $1/2000$, for $P$ it is $2/1000$, and for $T$ it is $1/1000$. The experiment $E$ is that a character is output and we observe (i) which type of device made that output and (ii) whether the character was correct. Write down the sample space. What is the probability the character was written on the disk, given $A = \{\text{it was incorrect}\}$?

16. A box contains 100 balls. 20 are red, 30 are green, and the rest are yellow. $3/4$ of the red balls are small (the rest are big), $2/3$ of the green balls are small, and $1/2$ of the yellow balls are small. The experiment is to choose a ball at random and to observe its color and its size.

(a) Describe the probability space for this experiment.
(b) What is the probability of the event $S = \{\text{a small ball is chosen}\}$?
(c) You are told that $A$ occurs. What is $P(R|S)$?

17. There are 4 envelopes, one of which contains 100 dollars, the other 3 being empty. You take one of the envelopes at random.

(a) What is the probability that you open it, it will contain 100 dollars?
(b) Now, before you open your envelope, somebody opens one of the other 3 and shows you that it is empty. You are now offered the choice to (i) keep your original envelope or (ii) change to one of the remaining 2. What is the probability you win 100 dollars if you do (i)? What is the probability if you do (ii)? Explain in detail.