1. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?

2. Suppose that it takes at least 9 votes from a 12-member jury to convict a defendant. Suppose also that the probability that a juror votes a guilty person innocent is 0.2, whereas the probability that the juror votes an innocent person guilty is 0.1. If each juror acts independently and if 65 percent of the defendants are guilty, find the probability that the jury renders a correct decision. What percentage of defendants is convicted?

3. There are two coins, one with probability $p_1$ of Heads and the other with probability $p_2$ of Heads. One of the coins is randomly chosen (with equal probabilities for the two coins). It is then flipped $n \geq 2$ times. Let $X$ be the number of times it lands Heads.
   
   (a) Find the PMF of $X$.
   
   (b) What is the distribution of $X$ if $p_1 = p_2$?
   
   (c) Give an intuitive explanation of why $X$ is not Binomial for $p_1 \neq p_2$ (its distribution is called a mixture of two Binomials).

4. A certain typing agency employees 2 typists. The average number of errors per article is 3 when typed by the first typist and 4.2 when typed by the second. If your article is equally likely to be typed by either typist, approximate the probability that it will have no errors.

5. Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$. Find the probability that 3 or more accidents occur today.

6. People enter a gambling casino at a rate of 1 every 2 minutes
   
   (a) What is the probability that no one enters between 12:00 and 12:05?
   
   (b) What is the probability that at least 4 people enter the casino during that time?

7. An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 are white. What is the probability that we shall make exactly $n$ selections?

8. There are three highways in the country. The number of daily accidents that occur on these highways are Poisson random variables with respective parameters 0.3, 0.5 and 0.7. Find the expected number of accidents that will happen on any of these highways today.

9. We have a well-shuffled deck of $n$ cards, labeled 1 through $n$. A card is a “match” if the card’s position in the deck matches the card’s label. Let $X$ be the number of matches; find $\mathbb{E}[X]$. 


10. In a group of $n$ people, what is the expected number of distinct birthdays among the $n$ people, i.e., the expected number of days on which at least one of the people was born? What is the expected number of birthday matches, i.e., pairs of people with the same birthday? (Assume that every day of the year is equally likely and that no one is born in a leap year)

11. A permutation $a_1, a_2, \ldots, a_n$ of $1, 2, \ldots, n$ has a local maximum at $j$ if $a_j > a_{j-1}$ and $a_j > a_{j+1}$ (for $2 \leq j \leq n-1$; for $j = 1$, a local maximum at $j$ means $a_1 > a_2$ while for $j = n$, it means $a_n > a_{n-1}$). For example, $4, 2, 5, 3, 6, 1$ has 3 local maxima, at positions 1, 3, and 5. For $n \geq 2$, what is the average number of local maxima of a random permutation of $1, 2, \ldots, n$, with all $n!$ permutations equally likely?

12. A couple decides to keep having children until they have at least one boy and at least one girl, and then stop. Assume they never have twins, that the “trials” are independent with probability 1/2 of a boy, and that they are fertile enough to keep producing children indefinitely. What is the expected number of children?

13. Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 inches$^2$ in $t$ minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3-second time interval.

14. Randomly, $k$ distinguishable balls are placed into $n$ distinguishable boxes, with all possibilities equally likely. Find the expected number of empty boxes.

15. A group of 50 people are comparing their birthdays (as usual, assume their birthdays are independent, are not February 29, etc.). Find the expected number of pairs of people with the same birthday, and the expected number of days in the year on which at least two of these people were born.

16. A total of 20 bags of Haribo gummi bears are randomly distributed to 20 students. Each bag is obtained by a random student, and the outcomes of who gets which bag are independent. Find the average number of bags of gummi bears that the first three students get in total, and find the average number of students who get at least one bag.

17. Two researchers independently select simple random samples from a population of size $N$, with sample sizes $m$ and $n$ (for each researcher, the sampling is done without replacement, with all samples of the prescribed size equally likely). Find the expected size of the overlap of the two samples.

18. There are 100 shoelaces in a box. At each stage, you pick two random ends and tie them together. Either this results in a longer shoelace (if the two ends came from different pieces), or it results in a loop (if the two ends came from the same piece). What are the expected number of steps until everything is in loops, and the expected number of loops after everything is in loops? 

   Hint: For each step, create an indicator r.v. for whether a loop was created then, and note that the number of free ends goes down by 2 after each step.

19. You have a well-shuffled 52-card deck. You turn the cards face up one by one, without replacement. What is the expected number of non-aces that appear before the first ace? What is the expected number between the first ace and the second ace?
20. There are $n$ prizes, with values $\$1, \$2, \ldots, \$n$. You get to choose $k$ random prizes, without replacement. What is the expected total value of the prizes you get?

21. A pizza parlor serves $n$ different types of pizza, and is visited by a number $K$ of customers in a given period of time, where $K$ is a Poisson random variable with mean $A$. Each customer orders a single pizza, with all types of pizza being equally likely, independent of the number of other customers and the types of pizza they order. Find the expected number of different types of pizzas ordered.