Problem 1 (10 points) Construct a sequence of operations such that when we are finished, the resulting Fibonacci heap is a path of length $2n$.

Problem 2 (20 points) Show that any sequence of $m$ MAKE-SET, FIND-SET, and UNION operations, where all UNION operations appear before any of the FIND-SET operations, takes only $O(m)$ time if both path compression and union by rank are used. What happens in the same situation if only the path compression heuristic is used?

Problem 3 (10 points) Let $G = (V, E)$ be a weighted, directed graph that contains no negative weight cycles. Let $s \in V$ be the source vertex, and let $G$ be initialized by INITIALIZE-SINGLE-SOURCE($G, s$). Prove that there is a sequence of $|V| - 1$ relaxation steps that produces $d[v] = \delta(s, v)$ for all $v \in V$.

(Hint: We stated in class that there is a shortest path tree and that the relaxation rule is $d[v] = \min\{d[v], d[u] + w(u, v)\}$)

Problem 4 (15 points) Prove or give a counterexample. For any graph $G$ with distinct positive weights associated with each cross edge and for any cut of $G$, the minimum weight spanning tree of $G$ contains exactly one edge belonging to the cut.

Problem 5 (10 points) Prove that if the edge weights of a graph $G = (V, E)$ are different, then the minimum weight spanning tree is unique.

Problem 6 (35 points) Let $G = (V, E)$ be a graph with $|V| = n$, $|E| = n + k$, where $k \ll \log n$ and edge weights $w_1, w_2, \ldots, w_{|E|}$. Give a fast algorithm to find a minimum weight spanning tree.