Next to each question, there is an indication of how hard we think each question is. Your answers should be as concise as possible while also fully explaining your solution. Please make an effort to write legibly.

1 (easy) Suppose that a root $x$ in a Fibonacci heap is marked. Explain how $x$ came to be a marked root.

2 (easy) Give a sequence of $m$ MAKE-SET, UNION and FIND-SET operations, $n$ of which are MAKE-SET operations, that takes $\Omega(m \log n)$ time when we use union by rank only.

3 (easy) Give an efficient algorithm to determine if an undirected graph is bipartite.

4 (hard) The diameter of a tree $T = (V, E)$ is given by

$$\max_{u,v \in V} \delta(u, v)$$

that is, the diameter is the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm. $|V|$ is the number of nodes in $T$.

5 (easy) Give a counterexample to the conjecture that if there is a path from $u$ to $v$ in a directed graph $G$, and if $d[u] < d[v]$ in a depth-first search of $G$, then $v$ is a descendant of $u$ in the depth-first forest produced.