

Nash eqib. is a solution concept (a condition which identifies the eqib) of a game involving two or more players, in which no player has anything to gain by changing only his or her own strategy unilaterally. In other words, if each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategies (choices and the corresponding payoffs (profits, rewards) constitute a Nash Eqarb. It is named after John Nash. In this case, each player either knows the strategies of all these players or can derive these.

lets first start with a two-player game (to be more formal)

some examples first:

Example ①: Prisoners' dilemma: two prisoners are on a trial for a crime and each one faces a choice of confessing to the crime or remaining silent. If they both remain silent, the authorities will not be able to prove charges against them and they will both serve a short prison term, say 2 years, for minor offenses. If only one confesses, he will get a reduced 1 year prison and he will be used as a witness against the other who will get 5 years. Finally if they both confess, they both will get a small break of cooperating with the authorities and will have to serve prison sentences of 4 years each (rather than 5). we can succinctly summarize the costs incurred in these four outcomes via the following two-by-two matrix, which is called a cost matrix because it contains the cost incurred by the players for each choice of their strategies.

	P_2 confess	P_2 silent
P_1 confess	4, 1	5, 5
P_1 silent	5, 4	2, 2

Expressing games in this form is the standard form or the matrix form is good for small # player and # strategies. however not good for big games thus we use the implicit form

note \rightarrow (confess, confess) is the only stable solution in each of the other three cases, at least one of the players can switch from silent to confess and improve his own payoff.

on the other hand the social optimum choice is the (silent, silent) case, which is not stable. Prisoners' Dilemma arise naturally in a lot of different situations with many players (see ISG games in chap 1 of the book of Nisan et al.)

Example ② we might have multiple outcomes which are stable
 Battle of sexes: consider two players, a boy and a girl, are deciding on how to spend their evening. They both consider two possibilities: going to a baseball game or going to a softball game: the boy prefers baseball and the girl prefers softball, but they both would like to spend the evening together rather than