Since games rarely possess dominant strategy solutions, we need to seek a less stringent and more widely applicable solution concept. (see e.g. the Battle of Sexes game)

Nash equib. captures the notion of a stable solution:

A strategy vector $s \in S$, is said to be a Nash equib. if for all players i and each alternate strategy $s'_i \in S_i$, we have that

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$$

Note that a solution is self-enforcing in the sense that once the players are playing such a solution, it is in every player's best interest to stick to his or her strategy.

Also clearly, a dominant strategy solution is a Nash Equib.

we may have several Nash equib. e.g. in the battle of sexes game.

★ <u>mixed strategy Nash Equib.:</u>

So far every thing was deterministic and thus called <u>pure strategy equib</u>

but Matching Pennies game did not possess any pure Nash equib.

If each player picks each of his two strategies with prob. $\frac{1}{2}$, then we obtain a stable solution in a sense, since the expected payoff of each player now is 0 and neither player can improve on this by a choosing a different randomization.

<u>To define randomized strategies formally</u>, let us enhance the choices of players so each one can pick a probability distribution over his set of possible strategies, such a choice is called a <u>mixed strategy</u>

we assume players independently select strategies using the probability distribution which leads to a probability distribution of strategy vectors s.

Thm (Nash 1951): Any game with a <u>finite set</u> of players and <u>finite set of</u> strategies has a Nash equib of mixed strategies.

If we do not have finite sets, there is not necessarily a mixed Nash (see the <u>pricing game</u> in the book chap.)

<u>The price of anarchy (PoA)</u> is the most popular measure of the inefficiency of equib. is defined as the ratio between the worst objective function value of an equi of the game and that of an optimal outcome (social optimum).

We are interested in a price of anarchy which close to 1, i.e., all equib. are good approximations of an optimal solution.