A game with multiple equi has a large price of anarchy even if only one of its equilibria is highly inefficient. The price of stability (PoS) is a measure of inefficiency designed to differentiate between games in which all equil are inefficient and those in which some equi is inefficient. Formally the price of stability of a game is the ratio between the best objective function value of one of its equilibria and that of an optimal outcome.

In games with a unique equi. PoA = PoS

A bound on PoS is much weaker than a bound on PoA

Two Reasons for PoS: 1- in some games a non trivial bound is possible only for PoS (PoA is very high)

2- if we envision the outcome as being initially designed by a central authority for subsequent use by selfish players then the best equi. is an obvious solution to the propose.

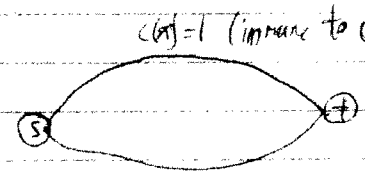
We see price of ~~Anarchy~~ Stability and more PoA in the next session.

In our class, we consider pure Nash Equi, which are Nash equi in which there is no randomization involved (vs. (mixed) Nash equi where the players are choosing their strategies randomly)

The inefficiency of equib cannot be bounded in general. a natural goal is to identify the classes of games in which equi are guaranteed to be approximately optimal (which happens for lots of classes)

In this session we consider selfish Routing.

Pigou's Example (1920)



Cost function c(e) describes the cost (e.g. travel time) incurred by users of the edge, as a function of the amount of traffic routed on the edge.

Suppose there is one unit of traffic, representing a very large population of players, and that each player chooses independently between the two routes from s to t. Assuming that each player aims to minimize its cost, the lower route is a dominant strategy