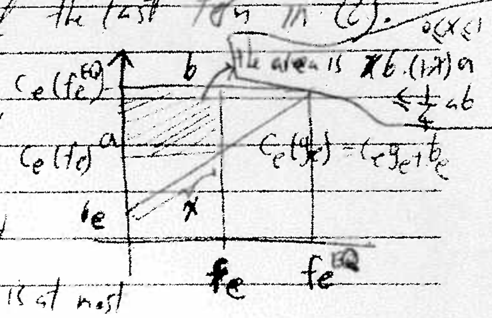
~~since the functions $c_e$ are non decreasing, we need to focus on the last expression for which $f_e < f_e^{EQ}$ to bound the last term in (2).~~

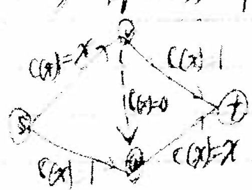
In this case $(c_e(f_e^{EQ}) - c_e(f_e))f_e$ is equal to the area of the shaded rectangle in Figure 1. Note that the area of any rectangle whose upper-left corner point is $(0, c_e(f_e^{EQ}))$ and whose lower-right corner point lies on the line corresponding $c_e(x) = c_e^{}x + b_e$ is at most half of the triangle defined by three points $(0, c_e(f_e^{EQ})), (0, b_e)$, and $(f_e^{EQ}, c_e(f_e^{EQ}))$. In turn, the area of the triangle is at most half of the rectangle by the two points $(0,0)$ and $(f_e^{EQ}, c_e(f_e^{EQ}))$. Thus




$$(c_e(f_e^{EQ}) - c_e(f_e))f_e \le \frac{1}{4} c_e(f_e^{EQ}) f_e^{EQ} \quad \text{as desired.}$$

Finally the result above can be generalized for different $c_e$ functions, i.e. when $c_e(x)$ is polynomial in $x$, with different constant independent of the network. See the book for more information.

Finally Braess' Paradox




For one unit of flow from s to t the equi is half-half with cost $\frac{3}{2}$.

Now assume we add $v \to w$ with $c_{vw}(f) = 0$.

Then the prev. Equi does not persist. the cost of new route $s \to v \to w \to t$ is never worse that along the two original paths, and it is strictly less whenever some traffic fails to use it. thus all flow uses the new path the cost would be 2 then. the optimum flow is the one before, i.e. $\frac{3}{2}$. so the price of anarchy is $\frac{4}{3}$. But as we proved above, since the price of anarchy is always at most $\frac{4}{3}$, it says adding edges to network with affine cost functions cannot increase the cost of equiflow by more than $\frac{4}{3}$ factor and thus Braess's Paradox is the worst case indeed.