... her initial endowment at these prices and buys her optimal bundle; the market clears, i.e., there is no deficiency or surplus of any good. An agent may have more than one optimal bundle; we will assume that we are free to give each agent any optimal bundle to meet the market clearing condition.

Note Fisher is a special case of Arrow-Debreu, $n+1$ goods and $m+1$ agents. Assume money is the $n+1$st good and for the first m agents correspond to the m buyers whose initial endowment is the money and the $m+1$st agent's initial endowment is all n goods. The first m agents have utilities for goods, as given by the Fisher market and no utility for money whereas the $m+1$st agent has utility for money only.

thm: there exists a market clearance prices in the Arrow-Debreu model and indeed it is unique.

See some ideas of the proof in the wireless network setting.

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proof of Thm A: First we observe that w.l.o.g each $q_j = 1$, by scaling the u_{ij}'s appropriately. u_{ij} and B_i are in general rational; again by scaling appropriately, we can assume they are integral.

Again similar to non-atomic selfish routing, we show equil. allocations for Fisher's linear case are indeed optimal solutions to a remarkable convex program, called the Eisenberg-Gale convex program.

The objective function in this program is not very intuitive though there are some evidence for it. The objective is

$$\max\left(\prod_{i\in B} u_i^{B_i}\right)$$

in some sense for each buyer i is $u_i^{B_i}$ is increased if u_i or B_i is increased. However we want buyers with larger B_i have higher effect in the system.

Exercise: Think more about this objective function

The log is used in Eisenberg-Gale convex prog.

Note that instead max $f(x)$ where $f(x)$ is concave we can min $-f(x)$, where $-f(x)$ is convex.

Note convex program min convex s.t. convex body (here is linear indeed)

$$\max \sum_{i\in B} B_i \log u_i$$

see the page after next page

s.t.

$$u_i = \sum_{j\in G} u_{ij} x_{ij} \quad \forall i\in B$$

$$\sum_{i\in B} x_{ij} \le 1 \quad \forall j\in G$$

$$x_{ij} \ge 0 \quad \forall i\in B, j\in G.$$