

Auction: we have a set of buyers and sellers in the market the more the situation becomes close to the perfect market scenario. An extreme opposite case is where there is only a single seller - an auction. The auction rules define the social choice, i.e. the identity of the winner. We have a set of alternatives (situations)  $A$  and a set  $n$  of players  $I$ . We have a valuation function  $v_i: A \rightarrow \mathbb{R}$ , where  $v_i(a)$  denotes the "value" that  $i$  assigns to alternative  $a$  being chosen. The value is in terms of some currency. We assume that if  $a$  is chosen and then player  $i$  is additionally given some quantity  $m$  of money, then  $i$ 's utility is  $u_i = v_i(a) + m$ , this utility being the abstraction of what the player desires and aims to maximize. Utilities of this form are called quasilinear preferences denoting the separable and linear dependence on money.

### Vickrey's Second Price Auction

we have a single item to sell and we have  $n$  players. Each player  $i$  has valuation  $v_i$  that he is "willing" to pay for this item. ~~if~~ If he wins the item, he has to pay some price  $p$  for it, then his utility is  $v_i - p$ , while if someone else wins the item then his utility is 0. Thus the set of Alternatives here is the set of possible winners,  $A = \{i - \text{wins} \mid i \in I\}$  and the valuation of each bidder  $i$  is  $v_i(i - \text{wins}) = v_i$  and  $v_i(j - \text{wins}) = 0$  for all  $j \neq i$ .

A social choice, i.e., an aggregation of the preferences of the different participants toward a single joint decision, is to allocate the item to the player who values it highest: choose  $i - \text{wins}$ , where  $i = \arg \max_j v_j$ . However we do not know all values, which are private, and we want to make sure that our mechanism decides on the allocation - the social choice - in a way that cannot be strategically manipulated. Our degree of freedom is the definition of the payment by the winner.

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