Consider some options:

- No payment: we give the item for free to the highest bid.
  It can be easily manipulated by exaggerating.
- Pay your bid. Again there is a problem.
  A player with value w_i who wins and pays w_i gets a total utility of 0. Thus he should declare a somewhat lower value $w_i' < w_i$ that still wins. He pays less w_i' and and his utility $u_i = w_i - w_i' > 0$.
  He essentially can bid the second price, if he knows that.

Here is the solution:

Vickery's second Price Auction: let the winner be the player i with the highest declared value of w_i and let i pay the second highest declared bid $p^* = \max_{j \neq i} w_j$.

<u>proposition (Vickery)</u> For every $w_1, \ldots, w_n$ and every w_i', let u_i be i's utility if he bids w_i and u_i' his utility if he bids w_i'. Then $u_i \geq u_i'$

<u>proof:</u> Assume that the valuation of i is w_i and the second highest value is p^*, then $u_i = w_i - p^* \geq 0$. Now for an attempted manipulation (reported) $w_i' > p^*$, i would still win if he bids w_i' and would still pay p^*, thus $u_i' =$
on the other hand, for $w_i' \leq p^*$, i would lose so $u_i' = 0 \leq u_i$
Now, if i loses by bidding w_i, then $u_i = 0$. let j be the winner in this case, and thus $w_j \geq w_i$. For $w_i' < w_j$, i would still lose and so $u_i' = 0 = u_i$
For $w_i' \geq w_j$, i would win but would pay w_j, thus his utility would be $u_i' = w_i - w_j \leq 0 = u_i$ $\square$

Thus this mechanism reliably computes a function (arg max) of n number (the w_i's) that are each held secretly by a different self-interested player.