Thm (VCG): every VCG mechanism is incentive

Truthful: $a = f(v_i, v_{-i})$

$v_i(a) + \sum_{j \neq i} v_j(a) - h_i(v_{-i})$

lies: $a' = f(v_i', v_{-i})$

$v_i(a') + \sum_{j \neq i} v_j(a') - h_i(v_{-i})$.

But since $a = f(v_i, v_{-i})$ maximizes social welfare over all alternatives

$V_i(a) + \sum_{j \neq i} V_j(a) \geq V_i(a') + \sum_{j \neq i} V_j(a')$ and thus the same inequality

holds when we subtract term $h_i(v_{-i})$ from both sides.

E.g. in Auction of a single Item, finding the player with highest value is exactly equivalent to maximizing $\sum_i V_i(i)$ since only a single player gets non-zero value. The Payment is clarke pivot payment below

If all $h_i = 0$, then though the mechanism is simple but the mechanism pays money.

Def: The choice $h_i(v_{-i}) = \max_{b \in A} \sum_{j \neq i} v_i(b)$

is called the clarke pivot payment. Under this rule the payment of player i is $P_i(v_1, \dots v_n) = \max_b \sum_{j \neq i} v_i(b) - \sum_{j \neq i} v_i(a)$

where $a = f(v_1, \dots, v_n)$.

Intuitively, i pays an amount equal to the total damage that he causes the other players. The difference between the social welfare of the others with and without its participation.

Lm: A VCG with clarke pivot payments makes the case that no players ~~always get non-negative utility~~ is ever paid.

Pf: Trivial

clarke pivot rules does not fit many situations where valuations are negative but it is fairly general rule payment.