Combinatorial Auctions: (A very general Auction Setting)

Def: A valuation v is a real-valued function that for each subset S of items, $v(S)$ is the value that bidder i obtains if he receives this bundle of items. A valuation must have free "disposal", i.e. be monotone: for $S \subseteq T$ we have that $v(S) \leq v(T)$ and it should be normalized: $v(\emptyset) = 0$.

usually we have subaddivity, i.e., $v(S \cup T) \leq v(S) + v(T)$

Again the utilities of bidders are "quasi-linear" in the money, i.e., if bidder i wins bundle S and pays a price of p for it then his utility is $v_i(S) - p$.

Also we assume that there are "no externalities", i.e., a bidder only cares about the item that he receives and not about how the other items are allocated among the other bidders.

Def: An allocation of the items among the bidders is $S_1, \dots, S_n$ where $S_i \cap S_j = \emptyset$ for every $i \neq j$. The social welfare obtained by an allocation is $\sum_i v_i(S_i)$. A socially efficient allocation (among bidders with valuations $(v_1, \dots, v_n)$) is an allocation with maximum social welfare among all allocations.

Note that if we use VCG payments, then these payments essentially charge each bidder his "externality": the amount by which his allocated bundle reduced the ~~total~~ reported value of the bundles allocated to the others. So this is incentive compatible.

but of course there are <u>some issues:</u>

- Computational complexity: The allocation problem is NP-hard even for simple cases
- Representation & Communication: The valuation functions are exponential sizes. how can we even represent them?