Sambuddha Roy

Research Statement

Introduction

My research interest lies in theoretical computer science with a primary focus on complexity theory and computational geometry. In complexity theory, we identify different classes of problems according to the amount of computational resources they require and study how those classes are related. The goal is to eventually get closer to answers to questions such as :

- ◇ "Is it inherently harder to find a proof than to verify it?" (This is the notorious P vs. NP question.)
- ◇ Does randomness fundamentally increase the power of algorithms, or can probabilistic algorithms all be made deterministic?"

While answers to these questions might not have an immediate impact on practical applications, they would profoundly enhance the understanding of a tremendously large number of problems.

On the other hand, looking at small space bounded language classes, a fundamental open question is to prove NL equal to L, that is to be able to eliminate the nondeterminism for logspace bounded machines. To restate the question, can we solve reachability in directed graphs in logspace? Can we at least solve reachability for restricted classes of (say, for instance, planar) digraphs in logspace?

In computational geometry, we are often faced with the onus of finding efficient algorithms (and combinatorial proofs) for theorems which are essentially topological in nature, like for instance, the Borsuk-Ulam Theorem. Looking for combinatorial proofs for some "discrete" manifestations of the Borsuk-Ulam Theorem like the Ham Sandwich Theorem and the Necklace Theorem will help us, not only in understanding the essential content of such theorems but also in constructing better algorithms for the related decision problems.

Thesis Research

My research has three broad themes, the latter two being similar in spirit :

- ◇ **Computational Complexity and Derandomization**
- ◇ **Interplay between Topology and Computation**
- ◇ **Algorithmic applications of Borsuk-Ulam**

Computational Complexity and Derandomization

One of the most active areas of research in computational complexity in the last decade was the development of sophisticated derandomization techniques. The idea is to build an algorithm, a Pseudo Random Generator (PRG) which makes crucial use of the insight that a resource bounded (randomized) algorithm will not be able to distinguish whether its (supposedly) random input is in fact random or not. Researchers (see for instance, [NW94]) constructed such generators from assumptions about the existence of functions which are "hard" to compute for circuits. Given that circuit lower bounds are hard to come by, the "easy-witness" technique of [IKW02] elucidated the issue that in fact to successfully derandomize some class of algorithms, circuit lower bounds are essential. These results were translated to a Kolmogorov setting by [ABK⁺02], thereby obtaining numerous interesting completeness results about Kolmogorov-random sets. In [AKRR03], my co-authors and I introduce new measures of resource-bounded Kolmogorov complexity, and compare the various measures against each other. We thus get a highly simplified picture of resource bounded Kolmogorov complexity measures and their relation to various circuit measures. For instance, the question as to whether certain pseudorandom generators exist (unconditionally) can now be equivalently stated as the question as to whether two different Kolmogorov measures are polynomially related.

In light of the widely-believed conjecture that NP $\neq$ P, researchers studied (the weaker question as to) whether NP can have polynomial size circuits instead. General belief prevails that any assumption leading to a collapse of the polynomial hierarchy PH is more likely to be untenable. Prior to our paper [CR06], others had proven that if NP has polynomial size circuits, then PH collapses to S_2^p, the symmetric alternation class. Along with Venkat Chakaravarthy, I introduce a new class O_2^p which is contained in S_2^p (and likely to be properly contained in S_2^p). One can imagine the class O_2^p as the culmination of the techniques of Sipser-Lautemann and Adleman. Using this, we improve a lot of the earlier collapses, the most prominent being : NP has polynomially sized circuits $\Leftrightarrow$ $\mathrm{PH} = \mathrm{O}_2^p$. We show thereby that this is in some sense the best collapse one can hope for. Among other results, we also prove improved collapses for [FPS03], [CCHO05], and we describe a new hierarchy which has properties similar to the polynomial hierarchy, PH.

Since most researchers believe that nondeterminism adds significant power to computations, it is natural to guess that NP-complete problems are truly tractable. However given the lamentable lack of lower bounds (in terms of the time complexity) for NP-complete problems, researchers looked at lower bounds on the runtime, given a limitation on the available space. Such results are called time-space tradeoffs. In [AKR⁺01], we prove such a time-space tradeoff for certain problems in the counting hierarchy.

Topology and Computation

This is joint work with Eric Allender and Samir Datta [ADR05b]. Ever since Barrington [Bar89] had proven the very surprising result that constant width polynomial size circuits capture exactly the languages in NC^1, researchers have been looking at topologically constrained constant width polynomial size circuits. Recently, Hansen in [Han04] proved that constant width polynomial size *planar* circuits capture exactly the circuit class ACC^0 (which is contained in NC^1). A natural question to ask then is how much more can we capture by increasing the genus of the underlying graph. It should not be too surprising to find out that constant genus constant width circuits still capture only ACC^0. But surprisingly, even when we allow polylogarithmic genus, we still get ACC^0. We are thereby led to compare other notions of non-planarity, and we find that while constant width circuits of *thickness* 1 capture ACC^0, thickness 2 gives a sharp increase in power to capture all of NC^1.

Afforded the intuition provided by the above that planarity/low genus can make computation simpler, we proceeded then to look at the question of whether reachability in directed *planar* graphs can be done in L. While Reingold [Rei05] recently showed in a breakthrough result that reachability in *undirected* graphs can be done in Logspace, the corresponding question for *directed* graphs has been open for three decades now. It is then natural to look for classes of directed graphs for which we can indeed solve reachability in Logspace.

In our paper [ADR05a], we prove that the problem of reachability in *planar* digraphs is closed under complementation, and also prove that a version of the problem can be done in the unambiguous logspace class, UL. This work is primarily topological in flavor. A surprising fact we discover is that reachability questions in directed *toroidal* graphs are no harder than reachability questions in planar graphs, while we could not adapt our technique to graphs of higher genus.

Grid graphs are a special type of planar graphs, where all the vertices are laid out on the lattice points of a grid, and all edges are laid out as grid edges. It might seem that reachability in directed grid graphs is somewhat simpler than reachability in directed planar graphs, but we prove that the two questions are logspace-equivalent. This would mean that when considering planar graphs, it suffices to focus on grid graphs.

In a follow-up paper to the above, along with Dave Barrington and Tanmoy Chakraborty [ABC^{+}05], we prove that reachability in a certain class of planar DAGs is in L. Namely, we prove that given a planar DAG which has only a constant number of sources (and an arbitrary number of sinks), we can answer reachability questions in such a graph in L. In fact, if the planar DAG is a grid graph and has only a single source and a single sink (and we call such a graph a SSGGR), we can prove that reachability questions in such graphs reduce to reachability in undirected grid graphs (a class which we call UGGR and which lies in but may *not* be complete for Logspace). This greatly generalizes results of [JLR01], where they give Logspace algorithms for series-parallel digraphs which are a special case of single source single sink planar DAGs. It might seem on the surface that the above questions are promise problems, but we also show how to recognize such graphs in Logspace. Cycles in directed graphs are usually hard to get around while working with a logarithmic-sized tape; nevertheless, we prove that reachability in outerplanar digraphs can also be done in L.

Algorithmic Applications of the Borsuk-Ulam theorem

Along with William Steiger [RS05], I look at some algorithmic questions related to the Borsuk-Ulam Theorem. The Borsuk-Ulam Theorem states that there cannot be a continuous antipodal map from S^n to S^{n-1}, where S^n is the sphere in $(n+1)$ dimensions. Over the years there have been lots of applications of this theorem from algebraic topology to combinatorics, at times giving rise to beautiful combinatorial theorems like the Ham Sandwich Theorem and the Necklace Theorem to cite a few. But most of the algebraic-topological methods are non-constructive, hence the interest in combinatorial proofs. We prove several upper and lower bounds for the algorithmic versions for such problems, and in some cases, these bounds are optimal. For example, given n points in the plane, it is easily seen that there always exists a pair of orthogonal lines dividing the plane into 4 sectors each of which has at most $\frac{n}{4}$ points. How difficult is it to find such a pair of orthogonal lines? We prove an optimal bound of $\theta(n \log n)$ for this problem. Following work by Barany and Matousek [BM01], Bereg [Ber04] recently proved, via a combinatorial argument, that given three point sets in the plane, there always exists a 2-fan bisecting each of the three point sets simultaneously. In fact, he shows that there are plenty of such bisecting 2-fans. Using his technique he gets an upper bound of $O(n \log^2 n)$ on the problem of finding such an "equitable" 2-fan. We show a lower bound of $\theta(n \log n)$. We also make progress on giving combinatorial proofs for some results related to the Necklace Theorem.

Research Plans

I plan to work in many areas of theoretical computer science and computational complexity, including derandomization, space-bounded computation, and computational geometry.

As exciting as current results in the context of derandomization and Kolmogorov complexity are, there are still some intuitively easy questions that are, surprisingly, still open. (For example, we cannot prove yet that the problem of deciding whether a string has high or low resource-bounded Kolmogorov complexity cannot be decided in polynomial time - even though this problem is complete for exponential time). Furthermore, it seems like there is some relationship between our approach and the notion of resource-bounded measure on complexity classes (as introduced by Lutz). It will be worthwhile to examine these questions in order to fully understand the versatility of our framework.

In the area of space-bounded computation, I would like to show that planar digraph reachability can indeed be performed in logspace. A first step in such a program would be to prove that planar DAG reachability is in logspace. Also, in light of the recent breakthrough by Reingold [Rei05] that undirected graph connectivity is solvable in logspace, there are numerous other questions (like for instance, "is the determinant of an integer matrix non-zero?") to which we might potentially apply such techniques.

In computational geometry, a daunting open question is to find constructive proofs for existential theorems, and thereby getting improved algorithms. Particular instances of such theorems waiting for constructive proofs abound, and I would like to investigate as to whether a unified approach of proving such results exists.

References

[ABC+05] E. Allender, D. Barrington, T. Chakraborty, S. Datta, and S. Roy. Grid Graph Reachability Problems. In *Electronic Colloquium on Computational Complexity*, 2005.

[ABK+02] E. Allender, H. Buhrman, M. Koucky, D. van Melkebeek, and D. Ronneburger. Power from Random Strings. In *IEEE Symposium on Foundations of Computer Science (FOCS)*, 2002.

[ADR05a] E. Allender, S. Datta, and S. Roy. The Directed Planar Reachability Problem. In *Proc. 25th annual Conference on Foundations of Software Technology and Theoretical Computer Science (FST&TCS)*, number 1373 in Lecture Notes in Computer Science. Springer, 2005. to appear.

[ADR05b] E. Allender, S. Datta, and S. Roy. Topology inside NC^1. In *IEEE Conference on Computational Complexity*, 2005.

[AKR+01] E. Allender, M. Koucky, D. Ronneburger, S. Roy, and V. Vinay. Time-space Tradeoffs in the Counting Hierarchy. *Electronic Colloquium on Computational Complexity (ECCC)*, 8(41), 2001.

[AKRR03] E. Allender, M. Koucky, D. Ronneburger, and S. Roy. Derandomization and Distinguishing Complexity, 2003.

[Bar89] David A. Barrington. Bounded-width polynomial-size branching programs recognize exactly those languages in NC^1. *Journal of Computer and System Sciences*, 38:150–164, 1989.

[Ber04] S. Bereg. Equipartition of Measures by 2-fans. In *International Symposium on Algorithms and Computation*, 2004.

[BM01] I. Barany and J. Matousek. Simultaneous Partitions of Measures by k-fans. *Discrete and Computational Geometry*, 2001.

[CCHO05] J-Y. Cai, V. Chakaravarthy, L. Hemaspaandra, and M. Ogihara. Competing Provers yield improved Karp-Lipton collapse results. *Inf. Comput.*, 198(1):1–23, 2005.

[CR06] V. Chakaravarthy and S. Roy. Oblivious Symmetric Alternation. In *Symposium on Theoretical Aspects of Computer Science (STACS)*, 2006.

[FPS03] L. Fortnow, A. Pavan, and S. Sengupta. Proving SAT does not have small circuits with an application to the Two Queries problem. In *IEEE Conference on Computational Complexity*, 2003.

[Han04] K. A. Hansen. Constant width planar computation characterizes ACC^0. In *Symposium on Theoretical Aspects of Computer Science (STACS)*, pages 44–55, 2004.

[IKW02] R. Impagliazzo, V. Kabanets, and A. Wigderson. In search of an easy witness: Exponential time vs. Probabilistic Polynomial time. *J. Comput. Syst. Sci.*, 65(4):672–694, 2002.

[JLR01] A. Jakoby, M. Liskiewicz, and R. Reischuk. Space Efficient Algorithms for Series-Parallel Graphs. In *18th International Symposium on Theoretical Aspects of Computer Science (STACS)*, number 2010 in Lecture Notes in Computer Science, pages 339–352. Springer, 2001. To appear in J. Algorithms.

[NW94] N. Nisan and A. Wigderson. Hardness vs. Randomness. *Journal of Computer and System Sciences*, 49:149–167, 1994.

[Rei05] O. Reingold. Undirected st-Connectivity in Logspace. In *Proceedings 37th Symposium on Foundations of Computer Science*, pages 376–385. IEEE Computer Society Press, 2005.

[RS05] S. Roy and W. Steiger. Combinatorial and Algorithmic Implications of the Borsuk-Ulam Theorem. In *Fall Workshop on Computational Geometry*, 2005.