

# A nonuniform lower bound for $NEXP$

Sambuddha Roy\*

June 27, 2005

## 1 Introduction

We try to find the best lower bound for  $NEXP$  that we can get given current techniques. We extend the results in [BHY94] and in [FK05].

## 2 A nonuniform lower bound for $NEXP$

We prove, extending [BHY94] that

**Theorem 1**  $NEXP \not\subset P^{NP[n^{o(1)}]}/n^{o(1)}$

**Proof:** To fix the ideas, we rather prove

**Proposition 2**  $NEXP \not\subset P^{NP[\log^2 n]}/\log$

We use translation lemmas and Kannan's result ([Kan82]) that there are languages in  $\Sigma_4^{exp}$  which do not have size  $2^{n/2}$  size circuits (say).

First, supposing that

$$NEXP \subset P^{NP[\log^2 n]}/\log,$$

and translating we get that

$$NEE \subset EXP^{NP[2^{\log^2 n}]} / poly$$

Now we have

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\*Department of Computer Science, Rutgers University, Piscataway, NJ 08855

**Lemma 3** For  $a(n) \geq \log n$ ,  $EXP^{NP[2^{a(n)}]} \subset DTime(2^{2^{a(n)}})_{\parallel}^{NP}$ .

The simulation to make all the adaptive queries nonadaptive is trivial.

We also have

**Lemma 4**  $DTime(2^{2^{a(n)}})_{\parallel}^{NP} \subset NTime(2^{2^{a(n)}})/2^{a(n)}$

This simulation is exactly similar to that of the proof that  $EXP_{\parallel}^{NP} \subset NEXP/poly$  (cf. for instance, [FK05]).

So from the supposition and the lemmas we have that  $NEE \subset NTime(2^{2^{\log^2 n}})/2^{\log^2 n}$ .

Another application of translation to the basic supposition yields  $NTime(2^{2^{\log^2 n}}) \subset DTime(2^{\log^2 n})^{NP}/2^{\log^2 n}$ . Altogether we have that  $NEE \subset DTime(2^{\log^2 n})^{NP}/2^{\log^2 n}$  which means that languages in  $NEE$  have quasipolynomial size  $SAT$ -oracle circuits. This contradicts Kannan.

The proof for  $n^{o(1)}$  adaptive queries, and  $n^{o(1)}$  advice follows similarly. This ends the proof of Theorem 1.  $\square$

## References

- [BHY94] B.Fu, H.Li, and Y.Zhong. An application of the translational method. *Mathematical Systems Theory*, 27:183–186, 1994.
- [FK05] Lance Fortnow and Adam Klivans. Np with small advice. In *Proceedings of the Twentieth Annual IEEE Conference on Computational Complexity (CCC)*. IEEE Computer Society Press, 2005.
- [Kan82] Ravi Kannan. Circuit-size lower bounds and non-reducibility to sparse sets. *Information and Control*, 55:40–56, 1982.