

Homework 4 Solutions

Q#22. (pg.266 : 10 points) Here, the size of the sample space is $|S| = 100$ and the number of integers in the range, that are divisible by 3 is the floor of $100/3$ i.e. 33. Hence, the desired probability is $33/100$.

Q#30. (pg.266 : 20 points) First of all, we try to figure out the size of the sample space, that is, the number of possible choices. The computer chooses 6 integers out of 40. (Here, we have to decide if the question means the integers chosen with or without replacement. We assume the integers to be chosen without replacement.) Now, we also ask ourselves if the order among the integers chosen matters. In this case, the order does not matter, hence, for purpose of computation of the size of the sample space, we are making choices of 6-combinations out of 40 integers. Hence, the total number of choices is ${}^{40}C_6$.

Now, we have to find out the probability of a winning combination. The player chooses another 6 integers, and wins only if exactly 5 of the integers chosen by the player are among the 6 chosen by the computer. In how many ways can we have this? First of all, we have to choose 5 integers out of the 6 chosen by the computer, and this we can do in 6C_5 ways. Now, for the last number chosen by the player, it has to be from the $(40 - 6)$ numbers not chosen by the computer, and can be done in ${}^{34}C_1$ ways. Hence, the total number of ways, is ${}^6C_5 \times {}^{34}C_1$. And the probability of the event of winning is :
$$\frac{{}^6C_5 \times {}^{34}C_1}{{}^{40}C_6}$$
.

Q#32. (pg.266 : 20 points) For two dice, the size of the sample space is just $6 \times 6 = 36$. The number of ways in which the two dice can add up to 8 is $(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)$, all in all, 5 possibilities. So, the probability of this event is $5/36$.

For three dice, the size of the sample space now is $6 \times 6 \times 6 = 216$. We have to find out the number of ways in which the three dice can sum up to 8. We frame the equation based on the numbers shown by each die.

$$x_1 + x_2 + x_3 = 8 \tag{1}$$

under the constraints $1 \leq x_1, x_2, x_3 \leq 6$. Because there are just three variables, and each is at least 1, hence, we don't need the upper constraint on each, it is automatically enforced. So the above system is equivalent to the same equation under the new constraints $1 \leq x_1, x_2, x_3$. And for this, we have that the number of solutions is ${}^{5+3-1}C_5$ (If we remember the way to solve such equations, we take 1 out of each variable, so that we now have $8 - 3 = 5$ balls to distribute in 3 boxes). Hence, the number of solutions is ${}^7C_2 = 21$. Therefore the probability is $21/216$.

So, the probability with two dice is higher.

Q#12. (pg.284 : 20 points) Call the two persons A and B . We are to find the probability that both A and B are born on the same day. We note that,

while the event specifies that they are to be born on the same day, it doesn't specify the exact date of birth. Consider the subproblem, which is to find the probability that both A and B are born on the same "particular" day, say, the 1st of January. This is just the probability that A is born on this day, multiplied (because of independence between A and B) by the probability that B is born on the same day. Hence, the probability comes out to be $1/366^2$. Now, the event that we are looking for that they are both born on the same day, but that day can be any day of the year, is just $366 \times \frac{1}{366^2} = \frac{1}{366}$.

Q#16. (pg.284: 20 points) To find the probability that we get 4 heads in a sequence of 5 flips, where the first flip is known to show tails. In that case, the only possible way to get the 4 heads is that all the other flips show heads. Hence, the probability of this event is just $(1/2)^4 = \frac{1}{16}$.

Q#18. (pg.284 : 10 points) Let E be the event that a bit string of length 3 has an odd number of 1's. Let F be the event that the string starts out with a 1. We list all the strings of length 3.

000
001
010
011
100
101
110
111

So, from inspection we have that $Pr(E) = \frac{4}{8} = \frac{1}{2}$, and also $Pr(F) = \frac{4}{8} = \frac{1}{2}$, and finally, $Pr(E \cap F) = \frac{2}{8} = \frac{1}{4}$. For independence, we require that $Pr(E \cap F) = Pr(E) \times Pr(F)$, which is actually obeyed here. So, the events are independent.