Problem 1
Let $k$ be any odd positive integer. We say that a binary string is $k$-monotone if it satisfies the following:

- its first bit (left most) is a 0.
- its last bit (right most) is a 1.
- The number of times we “transition” from 0 to 1, or 1 to 0, as we scan from left to right, is exactly $k$.

For example, 00010011100001 is a 5-monotone string of length 14 because it begins with a 0, ends with a 1, and as we scan from left to right (beginning with the left most bit), we transition from 0 to 1 when we move from bit 3 to bit 4 (if we label the leftmost bit as position 1), transition again from 1 to 0 when we move from bit 4 to bit 5, transition from 0 to 1 again as we move from bit 6 to 7, transition again from 1 to 0 as we go from bit 9 to 10, and finally transition from 0 to 1 as we transition position 13 to 14.

Find the number of 11-monotone strings of length 1000.
**Hint:** Try to think about how these strings looks like.

Problem 2
We say that a sequence $(0, y_1, \ldots, y_{10}, 100)$ is increasing if $y_1, \ldots, y_{10}$ are integers and

$$0 < y_1 < y_2 < \ldots < y_{10} < 100.$$

Find the number of increasing sequences of the above form.
**Hint:** Try and see if you can relate this problem to the solution of the books problem from the lecture.

Problem 3
Find all integer solutions to

$$3x_1 + x_2 + x_3 + x_4 \leq 10,$$

under the constraint that $x_1, x_2, x_3, x_4 \geq 1$.
**Hint:** Partition the set of all solutions into parts based on the value of $x_1$. 