Problem 1

Three fair dice colored red, blue and green are rolled.

1. What will you choose the outcomes and sample space to be in this case?
2. What is the total number of events in the sample space? How many atomic events are there in the sample space?
3. What probability measure will you use to model the process of rolling the dice?
4. What is the probability that exactly two of the dice roll the same number?
5. What is the probability that all three dice roll distinct numbers?
6. What is the probability that at least two of the dice roll the same number?

Give explanations for all your answers and show all the steps involved.

Problem 2

You are at a casino table and are about to play a game. The rule of the game is as follows: at each step a fair coin is tossed and if it comes up heads you win $1, and if it comes up tails you lose $1. The game consists of 50 such steps.

1. What will you choose the outcomes and sample space to be in this case? What is the size of the sample space?
2. What probability measure on the sample space will you use to model this random process?

Define your net gain to be the total amount of money won by you during the game minus the total amount of money lost by you during the game.

1. Consider the event
   \[ E_1 = \text{Your net gain is zero at the end of the game.} \]
   How will you write \( E_1 \) as a subset of \( \Omega \)?

2. What is the probability that you end up with zero net gain?
3. Consider the event
   \[ E_2 = \text{Your net gain is positive at the end of the game.} \]
   How will you write \( E_2 \) as a subset of \( \Omega \)?

4. What is the probability that you end with a positive net gain?