Problem 1
Suppose a black dice and a white dice and rolled together. Use the partition method to find the number of possible outcomes where the number that turns up on the black dice is strictly larger than the number that turns up on the white dice.

Problem 2
Suppose a black dice and a white dice and rolled together. Find the number of possible outcomes where the number that turns up on the black dice is at most the number that turns up on the white dice.

Problem 3
How many numbers are there between 1 and 100 that are either prime or multiples of 6?

Problem 4
Suppose a black dice and a white dice and rolled together. Find the number of possible outcomes where the product of the numbers on the two dice is odd.
Hint: Let \( A \) be the set of all outcomes where the product of the two numbers is odd. Try to partition \( A \) into parts based on the number on the black dice.

Problem 5
Suppose a black dice and a white dice and rolled together. Find the number of possible outcomes where the sum of the numbers on the two dice is not divisible by 7.
Hint: Difference method

Problem 6
Use the partition method to show that the total number of possible outcomes when three dice are rolled (a white, a black, and a red dice) is 216.
Hint: Use a 3-tuple \((b, r, w)\) to model an outcome. The set of all outcomes, then, is \( A \times A \times A \), where \( A = \{1, \ldots, 6\} \).
Now partition \( A \times A \times A \) into 6 parts: \( A_1, \ldots, A_6 \) such that \( A_i \) is the set of all outcomes where the black dice turns up the number \( i \). What is \( |A_i| \)? Use that along with the sum rule to get the answer.
Problem 7

A binary string is called \textit{balanced} if it has equal number of ones and zeros. Let $S$ be the set of balanced binary strings of length 14, and let $P$ be the set of paths from $A$ to $B$ in the grid below that use only downward and rightward steps (i.e., a path can never go up, left, or diagonal; only down or right). Show that $|P| = |S|$.