Problem 1

How many distinct permutations are there of the word “ENGAGE”?

**Hint:** First, begin by “disambiguating” the duplicates, i.e. consider the word “$E_1NG_1AG_2E_2$”. Let $A$ be the distinct permutations of “$E_1NG_1AG_2E_2$” and $B$ the set of all distinct permutations of “ENGAGE”. Use the division rule to relate $|B|$ to $|A|$ and then find $|B|$.

Problem 2

Suppose two identical dice (both white) are rolled. How many outcomes are there? Is the number of outcomes in this case same as the number in the case when one of the dice is white and the other is black? How would you represent an outcome in the case of two identical dice?

Problem 3

Suppose you have 2 identical white beads, a blue bead, a read bead, a green bead, and a yellow bead. You also have a string that you can pass through all the beads to form a necklace with 6 beads. Two necklaces are said to be identical if every bead has the same left-neighbor and the same right-neighbor in both necklaces. In other words, if you can rotate one necklace to match the other, they are considered identical. Find the number of distinct necklaces you can form using the 6 beads.

**Hint:** You can think of this as the same problem as seating knights around the round table except that this time there are two identical and indistinguishable knights (basically, the two white beads). First assume that the two white beads are distinct (by assuming there is a mark on one of them, etc.), count the number of ways of making a necklace in this case, and then relate this case to the case when the two white beads are identical.

Problem 4

1. How many strings are there of length 10 that contain seven zeros, one 1, one 2, and one 3?

2. Use the answer from part 1 along with the division rule to find the number of binary strings of length 10 with seven zeros and three ones.

3. Can you use this approach to find the number of binary strings of length $n$ with $k$ ones and $n - k$ zeros?

   **Hint:** Let $B$ be the set of all binary strings of length $n$ with $k$ ones and $n - k$ zeros. Let $A$ be the set of all strings of length $n$ that can be made using $n - k$ ones and some $k$ distinct characters (which are all distinct from 0, and each other) $c_1, c_2, \ldots, c_k$. Can you relate $|B|$ to $|A|$ by generalizing what you did in part 2 using the division rule? Also, use your ideas from part 1 to compute $|A|$ and then use that to find $|B|$.