Quiz I [Solution] (CS 206 - Spring 2020) [2 problems, 20 pts, 20 mins]

Name:
NetID:

This is an open notes quiz; you may use any printed or hand-written material. The use of any electronic devices during the quiz or other attempts at cheating will have dire consequences.

1. (5 × 2 pts each = 10 pts) State which of the following are true/false. You don't need to explain/prove your answers.
   
   (a) If $A = \{1, 2, 3\}$ then $\{(2, 3, 1), (1, 2, 3)\} \subseteq A^3$.
   True. $A^3$ consists of all sequences of length 3 that can be made using 1s, 2s, and 3s. $(2, 3, 1), (1, 2, 3)$ are both such sequences and so $\{(2, 3, 1), (1, 2, 3)\} \subseteq A^3$.
   
   (b) Let $A = \{1, 2, 3\}$ and $B \subseteq A^2$ such that $B = \{(b, w)| b + w = 5\}$ then $|B| = 3$.
   False. $B = \{(2, 3), (3, 2)\}$ and so $|B| = 2$.
   
   (c) The function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined as $f(n) = n + 1$ is a bijection. False. There is no element that is mapped by $f$ to the 0 in the codomain.
   
   (d) Let $A \subseteq S$ and suppose $A$ and $S$ are finite sets. Then $|A| = |S| - |S - A|$.
   True. This is the formula we use when applying the difference method.
   
   (e) $\{1, 2, 3\} \neq \{2, 1, 3\}$.
   False. Sets do not have an inherent order to their elements and so changing the order in which the elements are enumerated does not change the set.

2. (10 pts) Suppose a black dice, a white dice, and a red dice are all rolled together. How many outcomes are there in which the number on the black dice is equal to the number on the white dice? Explain your solution in a brief manner.
   
   Solution: We can model the outcomes using 3-tuples/sequences of length 3 where each element is a number between 1 and 6, and the first element is the number that comes up on black dice, the second element is the number that comes up on the white dice, and the third element is the number that comes up on the red dice. Consider the set $S$ defined as
   
   $$S = \{(b, w, r)| b = w\}.$$ 
   
   Let us partition $S$ into 6 parts $S_1, \ldots, S_6$ such that
   
   $$S_i = \{(b, w, r)| b = w = i\}.$$ 
   
   For each $1 \leq i \leq 6$, it is easy to see that $|S_i| = 6$. This is because the only outcomes where $b = w = i$ are $(i, i, 1), (i, i, 2), \ldots, (i, i, 6)$. Using the partition method, it follows that
   
   $$|S| = \sum_{i=1}^{6} |S_i| = 6 \times 6 = 36.$$