Sample midterm exam

CS 206: Discrete Structures II
Spring 2020

Total points: 10
Duration: 1 hour

Name: _______________________
NetID: ______________________

INSTRUCTIONS:

1. You have to solve 6 problems in 1 hour. To get full points for a problem, you must give details for all the steps involved in your solution AND arrive at the correct answer. Giving partial details or arriving at the wrong answer may result in a partial score.

2. You may leave your answer in terms of factorials, binomial coefficients, and/or powers of numbers.

3. Make sure you write your solutions ONLY in the space provided below each problem. There is plenty of space for each problem. You can use the back of the sheets for scratchwork.

4. You may refer to physical copies of any books or lecture notes during the exam. However, the use of any electronic devices will lead to the cancellation of your exam and a zero score, with the possibility of the authorities getting involved.

5. Make sure you write your name and NetID in the space provided above.

6. If we catch you cheating, or later suspect that your answers were copied from someone else, you will be given a zero on the exam, and might even be reported to the authorities!
Problem 1. [10 pts]
A password can consist of lower case letters (a-z) and digits (0-9). How many passwords of length 10 are there than contain 3 ‘a’s, 3 ‘0’s, 2 ‘g’ and 2 ’1’s and all the digits appears after the lower case letters?
Problem 2. [10 pts]
There are $n$ bulbs in a row. All the bulbs are turned off to begin with. Our goal is to get to the state when all the bulbs are turned on. At every step, we are allowed to turn exactly one bulb on. No bulb that has been turned on can be turned off again. In how many ways can we go from the initial state (all bulbs off) to the final state (all bulbs on)?
Problem 3. [20 pts]
Consider functions of the form $f : \{1, \ldots, 10\} \to \{1, \ldots, 10\}$ such that for all $1 \leq x \leq 10$ it must be the case that $f(x) \leq x$. Find the number of such functions.
Problem 4. [20 pts]
A password can consist of lower case letters (a-z) and digits (0-9). How many passwords of length 10 are there that contain at least one letter and at least one digit?
Problem 5. [20 pts]
Suppose we want to form a committee consisting of 10 members such that one of the 10 members is appointed as the chairman and another is appointed as the deputy-chairman. Furthermore, the committee must have at least one woman. If we can choose from 20 men and 20 women, how many ways are there of forming the committee and appointing the chairman and deputy-chairman?
Problem 6. [20 pts]
Suppose the setup and the requirements are exactly same as the previous problem except that there is an additional requirement that the chairman must be a woman. How many ways are there to form the committee and do the appointments?