Quiz I (CS 205 - Fall 2019)

Name:

NetID:

Section No.:

For each of the following problems, use the space provided below the problem statement to write down your answer. Write clearly and concisely. There are 4 problems in total.

1. (20 pts) Consider an argument whose premise is $\exists x (P(x) \rightarrow Q(x))$ and conclusion is $(\forall x \neg Q(x)) \rightarrow \neg (\forall x P(x))$. Prove that this is a valid argument using rules of inference. Show all the steps and mention the rule you use in each step.

   (Hint: What can you do when the conclusion of an argument is of the form $p \rightarrow q$?)

2. (5 + 5 = 10 pts) Write the following propositions in the “If... then ...” form:

   (a) “A sufficient condition for $NP$ to be contained in $BPP$ is that SAT can be solved in randomized polynomial time”.
(b) “A necessary condition for Graph isomorphism to be $NP$-complete is that the polynomial hierarchy collapses”.

3. (10 pts) Suppose the domain of discourse is the set of integers and let $P(x)$ be the predicate “$x$ is a perfect square”. Express $P(x)$ as a predicate formula using logical connectives, quantifiers, and other mathematical symbols (if needed).
(Recall: $x$ is a perfect square if $x$ can be written as $x = n^2$ for some integer $n$.)

4. (10 pts) Is the proposition $p_1 \land p_2 \land \ldots \land p_{1001} \land (p_1 \oplus p_2 \oplus \ldots \oplus p_{1001})$ a contradiction?
Provide a short explanation for your answer.
(Recall: A contradiction is a proposition that is always false, regardless of what truth values are assigned to the variables.)